

The forms of half-integrals and half-derivatives for polynomials of the form

$$P_n(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

$$\begin{aligned} \left(J^{\frac{1}{2}} P_n \right) &= \frac{1}{\Gamma(\frac{1}{2})} \sum_{i=0}^n \frac{2^{i+1} a_i i! x^{\frac{2i+1}{2}}}{(2i+1)!!} \\ \left(\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}} P_n \right) &= \frac{1}{\Gamma(\frac{1}{2})} \sum_{i=0}^n \frac{2^i a_i i! x^{\frac{2i-1}{2}}}{(2i-1)!!} \end{aligned}$$

By way of the identity

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!},$$

we can calculate the half-integral and half-derivative of the exponential function:

$$\begin{aligned} \left(J^{\frac{1}{2}} e^x \right) &= \frac{1}{\Gamma(\frac{1}{2})} \sum_{i=0}^n \frac{2^{i+1} a_i x^{\frac{2i+1}{2}}}{(2i+1)!!} \\ \left(\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}} e^x \right) &= \frac{1}{\Gamma(\frac{1}{2})} \sum_{i=0}^n \frac{2^i a_i x^{\frac{2i-1}{2}}}{(2i-1)!!} \end{aligned}$$